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Engineering Notes

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Longitudinal Long-Term Modes in Super- and Hypersonic Flight

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Introduction

IN recent papers,[1–13] the dynamics of the aircraft is treated with particular reference to the modes of motion. Emphasis is placed on approximate formulas for describing the modes. Formulas of different degrees of complexity are presented so that a great variety of vehicle characteristics and/or airplane types can be accounted for. This renewed interest into aircraft modes has produced valuable results, yielding more accurate approximate solutions for describing the effects of vehicle parameters or an improved insight and understanding of longitudinal and lateral flight dynamics.

One mode is the well-known phugoid for which several approximations including new expressions are given. An interesting aspect with regard to phugoid characteristics concerns the super- and hypersonic flight regimes that show unique relationships.

Another aspect relates to the fact that there is a third longitudinal mode termed height mode. This mode exists in super- and hypersonic flight. The height mode is an essential constituent of aircraft dynamics in the high-speed region addressed.

Therefore, it is the purpose of this Note to derive and present the properties of the phugoid and the height mode that form the longitudinal long-term modes in super- and hypersonic flight. The unique features of the phugoid are described, showing fundamental differences when compared with the conventional speed regime of subsonic flight. Furthermore, the height mode is considered in detail, and its significance for the dynamics of aircraft is substantiated.

Basic Properties of Long-Term Modes

With reference to the equations of motion accounting for altitude-dependent forces and moments (e.g., Refs. 14 and 15), the following relations can be derived for the phugoid by approximate factorization of the characteristic equation:

$$\omega_{np} \approx \sqrt{-g\rho_h[1 + (2 - \beta_{ML})k_\rho]}$$

$$\zeta_p\omega_{np} \approx [(1 - \beta_{ML}/2)(1 - n_h) - (k_\rho/2)n_V](C_D/C_L)g/V_0 \qquad (1)$$

In these relations, the influence of the curvature of the Earth on the dynamics of the vehicle is not considered (becoming effective for hypersonic Mach numbers greater than about $M = 8$).

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From Eq. (1) it follows that there are two quantities exerting an influence on the frequency ω_{np} of the phugoid, i.e., ρ_h and k_ρ. These quantities, which are related to the atmospheric density and the speed, are given by

$$\rho_h = \frac{1}{\rho}\frac{d\rho}{dh}, \qquad k_\rho = -\frac{g/\rho_h}{V_0^2} \qquad (2)$$

As shown in Fig. 1, k_ρ is very small for Mach numbers greater than $M = 2$ at all altitudes of interest. The factor β_{ML} that is used to describe the dependency of the lift coefficient on the Mach number reads for supersonic flight according to the Ackeret rule

$$\beta_{ML} = M_0^2 / \left(M_0^2 - 1\right) \qquad (3a)$$

This factor reduces the influence of k_ρ, particularly in the low supersonic Mach number region at $M = 1.5$. In hypersonic flight, the lift coefficient is independent of the Mach number so that β_{ML} can be set to

$$\beta_{ML} = 0 \qquad (3b)$$

Accounting for Eqs. (3a) and (3b) and for the smallness of k_ρ (Fig. 1), the relation for the phugoid frequency in super- and hypersonic flight ($M > 1.5$) may be rewritten as

$$\omega_{np} \approx \sqrt{-g\rho_h} \qquad (4)$$

From this relation, it follows that the phugoid frequency is practically determined solely by the atmospheric density gradient ρ_h. Inspection of the data of the atmosphere[16] reveals that ρ_h shows comparatively small changes in altitudes up to 80 km (the range considered for k_ρ in Fig. 1). With values less than ±3%, the ρ_h changes are particularly small in the altitude region between 11 and 35 km, which may be especially of interest for super- and hypersonic flight. As a result, the phugoid frequency shows correspondingly small changes (this being additionally supported by the fact that the effect of the small ρ_h changes on ω_{np} is further reduced because of the square root dependence of ω_{np} on ρ_h). The described result is confirmed by a numerical evaluation presented in Fig. 2, using the complete set of the linearized equations of motion. The results, which cover a wide range of Mach numbers and altitudes relevant for supersonic flight, show that the phugoid frequency is at about

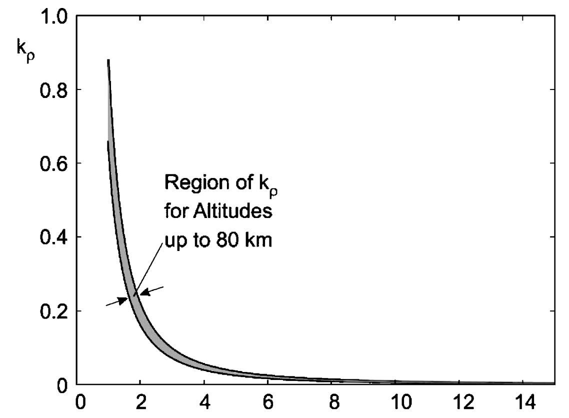

Fig. 1 Factor k_ρ (atmospheric data from Ref. 16).

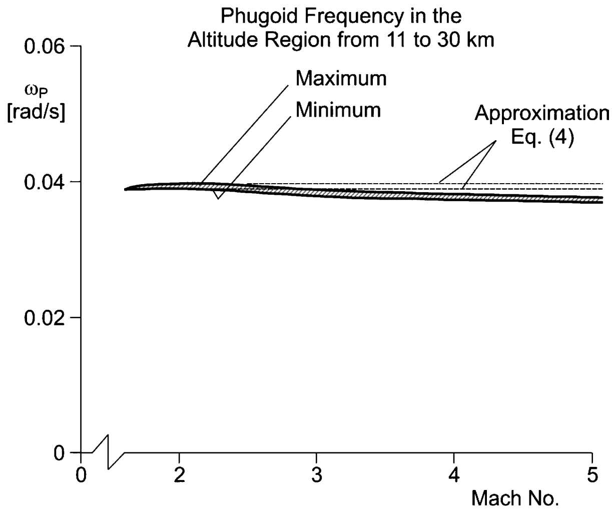

Fig. 2 Phugoid frequency of a supersonic aircraft.

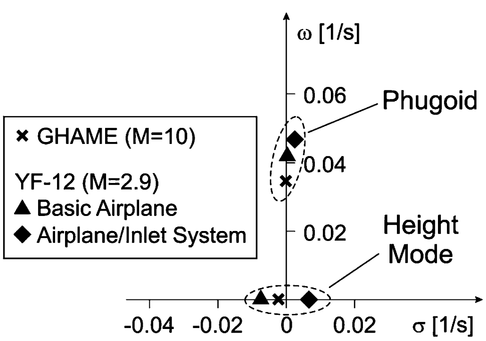

Fig. 3 Phugoid and height mode characteristics of aircraft at super- and hypersonic speeds, obtained in flight tests (YF-12; Ref. 17) and from computation (GHAME; Ref. 14).

0.04 rad/s. Further evidence of the described phugoid frequency property is provided by the experimental and computational data presented in Fig. 3. The YF-12 data are obtained in flight tests,[17] and the GHAME data are from a numerical computation with a full dynamics modeling (GHAME model; Ref. 18).

The preceding result implies that the phugoid frequency does practically not depend on speed. This is a fundamental difference to the well-known properties of the phugoid in the conventional speed region where its frequency is proportional to the inverse of the speed $\omega_{np} \approx \sqrt{(2)}g/V_0$.

As regards the damping ζ_p of the phugoid, it follows from Eq. (1) that there are two effects due to n_V and n_h that describe the dependencies of the thrust on speed and altitude according to

$$n_V = \frac{V_0}{T_0}\frac{\partial T}{\partial V}, \qquad n_h = \frac{1}{\rho_h T_0}\frac{\partial T}{\partial h} \tag{5}$$

In super- and hypersonic flight, it can be assumed for airbreathing engines that $n_h \approx 1$ (which strictly holds for an altitude range with constant temperature, as in the stratosphere) and $0 \leq n_V \leq 2$ (Ref. 19). Using these values of n_V and n_h and accounting for the smallness of k_ρ (Fig. 1), Eq. (1) yields for the damping of the phugoid

$$\zeta_p \approx 0 \tag{6}$$

As a result, there is practically no damping of the phugoid. This result is also confirmed by the data presented in Fig. 3, for the values from flight tests as well as from computations. A more detailed treatment supporting the described result is given in Ref. 20.

There is again a substantial difference concerning the conventional speed region. This is because the dependence of thrust on speed exerts a significant influence on the damping of the phugoid in the conventional speed region. By contrast, no such effect exists in super- and hypersonic flight. Furthermore, the dependence of thrust on altitude, described by $n_h \propto \partial T/\partial h$ in Eq. (1), has a significant effect in super- and hypersonic flight, but is without any influence in the conventional speed region.

For the height mode, the following relation valid for super- and hypersonic flight can be derived:

$$s_h \approx (1 - k_\rho)[n_V - (2 - \beta_{ML})n_h](C_D/C_L)g/V_0 \tag{7}$$

This relation shows as a basic result that the height mode is primarily determined by thrust characteristics. In terms of n_V and n_h, the thrust/speed dependency is of similar significance as the thrust/altitude dependency. An increase of thrust with speed ($n_V > 0$), a possible engine characteristic in supersonic flight,[19] leads to a destabilization of the height mode. A decrease of thrust with altitude ($n_h > 0$), which corresponds with the usual behavior of airbreathing engines, yields a stabilizing effect.

Because s_h is inversely proportional to the lift-to-drag ratio and to the speed, it is numerically small. Its magnitude manifests in the data presented in Fig. 3 obtained in flight tests and numerical computations.

The preceding results show that there is a unique effect of the thrust/altitude dependency on aircraft stability. It means that the thrust/altitude dependency principally causes a destabilization, independent of the sign of n_h, be it positive or negative. This is because its effects on the phugoid and on the height mode are opposite to each other [Eqs. (1) and (7)].

Effect of Pitching Moment Properties on Long-Term Modes

Pitching moments due to speed and altitude exert a significant influence on the phugoid and the height mode. These pitching moments, which may be due to aeroelastic deformations, to compressibility effects, or to thrust-axis offset with respect to the center of gravity, can be described by the stability derivatives C_{mV} and C_{mh}.

As regards the speed dependent moment, $C_{mV} = \partial C_m/\partial(V/V_0)$, the following relations can be derived for the phugoid and the height mode:

$$\omega_{np} \approx \omega_{np}^* \sqrt{1 - k_\rho(C_{L\alpha}/C_L)C_{mV}/C_{m\alpha}}, \qquad \zeta_p \approx \zeta_p^*$$

$$s_h \approx s_h^* + (C_{D\alpha}/C_L)(g/V_0)C_{mV}/C_{m\alpha} \tag{8}$$

where the quantities with an asterisk denote the case $C_{mV} = 0$ given by Eqs. (1) and (7).

From Eq. (8) it follows that the phugoid is influenced only to a small extent, which is further reduced with increasing speed because k_ρ is proportional to $1/V_0^2$. This is confirmed by results of a numerical computation presented in Fig. 4. The small effect of speed-dependent moments again contrasts with the relationships in the conventional speed regime, which shows a strong influence of C_{m_V}.

Other than the phugoid, the height mode is significantly influenced by the speed-dependent moment. A positive C_{m_V} value yields a stabilizing effect, whereas the opposite holds for a negative value. As regards the conventional speed region, there is no such an effect because of the non-existence of the height mode.

The effect of an altitude dependent moment, $C_{mh} = (1/\rho_h)\partial C_m/\partial h$, can be described by the following relations (where $n_h = 1$):

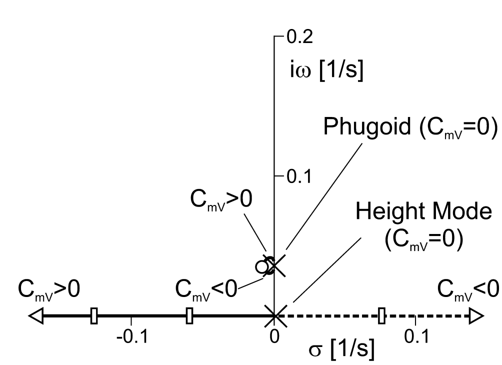

Fig. 4 Effect of speed-dependent moment on phugoid and height mode in hypersonic flight ($M = 10$).

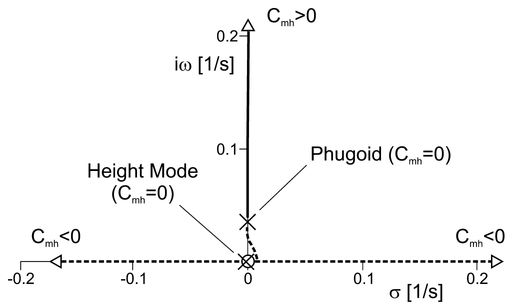

Fig. 5 Effect of altitude-dependent moment on phugoid and height mode in hypersonic flight (*M* = 10).

$$\omega_{np} \approx \omega_{np}^* \sqrt{1 - (C_{L\alpha}/C_L)C_{mh}/C_{m\alpha}}$$
$$2\zeta_p\omega_{np} \approx 2\zeta_p^*\omega_{np}^* + [C_{D\alpha}/(C_L C_{m\alpha} - C_{mh})](g/V_0)C_{mh}$$
$$s_h \approx s_h^* - [C_{D\alpha}/(C_L C_{m\alpha} - C_{mh})](g/V_0)C_{mh} \tag{9}$$

There is a strong effect of altitude-dependent moments on the phugoid frequency, yielding an increase for positive C_{mh} values and a decrease in the opposite case. For C_{mh} values more negative than

$$(C_{mh})_{\text{crit}} \approx C_L C_{m\alpha}/C_{L\alpha} \tag{10}$$

the phugoid is even changed from an oscillation into an aperiodic mode. This is accompanied with dynamic instability, yielding an aperiodically unstable mode. The described effects are confirmed by the numerical results presented in Fig. 5. Figure 5 also shows that the changes in the damping of the oscillatory phugoid and the height mode are smaller.

There is again a fundamental difference to the relationships in the conventional speed region. This is because the effects of altitude-dependent moments are unique for super- and hypersonic flight, and there is no such influence in the conventional-speed region.

Conclusions

The longitudinal long-term modes of motion of aircraft in super- and hypersonic flight are considered. They consist of the phugoid and an additional mode, which is termed height mode, that does not exist in the conventional-speed regime. Approximate formulas for both modes are derived. The unique properties of the phugoid in super- and hypersonic flight are described, and it is shown that there are fundamental differences when compared with the conventional-speed region. Furthermore, the properties of the height mode are presented, and its significance for aircraft dynamics is substantiated. The findings for the phugoid and the height mode are confirmed by results obtained in flight tests and by solutions for the complete equations of motion. The unique properties of the phugoid and the height mode particularly concern the substantial effects of altitude-dependent forces and moments as well as the reduction of the speed related influence.

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